项目编号：T426PRP43001

上海交通大學

本科生研究计划（PRP）研究论文

（第 43 期）

论文题目：基于 SHINE 装置的缪子源研究-SXFELμ-探测方案研究

项目负责人：王量 学院（系）：物理与天文学院

指导教师：Kim Siang Khaw 学院（系）：李政道研究所

参与学生：刘方超，陈思元，王量

项目执行时间：2023 年 2 月至 2024 年 2 月

基于 SHINE 装置的缪子源研究

物理与天文学院 F2107004 王量

指导教师： 李政道研究所 Kim Siang Khaw

摘要

本文首先介绍缪子重要性以及建造缪子源的动机，之后提出建设基于 SHINE(Shanghai HIgh repetitioN rate XFEL and Extreme light facility)和 SXFEL(Shanghai Soft X-ray Free-Electron Laser User Facility)装置废束桶建立低成本缪子源三步走计划，这篇文章介绍负缪子探测。首先用 Geant4 对比 MIXE(Muon-Induced X-ray Emission)和相应结果进行不同能量区间入射 negative muon 模拟实验的可靠性验证。之后基于简单装置优化高能缪子入射的 Degrader 选材以使得我们得到适应我们探测器分辨率的谱线，保证足够的事例数。再后考虑厚度优化，Al 板不能过薄使得低能电子穿透，过厚使得高能电子产生大量韧致辐射产生巨大的本地，所以 5mm-9mm 的铝板作为 Degrader 最优。随后进行基于 SXFEL 全流程模拟找到最高信噪比的探测装置摆放位置并且给出该位置相应的μ/γ比用于通过接收器接收到的光子推断缪子产额。最后介绍相应 DAQ(Data Acquisition)系统以及为下一阶段 SXFEL 实验的准备。

关键词：geant4 模拟，musrSim 模拟，缪子源，负缪子

ABSTRACT

This article begins by introducing the importance of muons and the motivation for constructing a muon source. It then proposes a three-step plan to establish a low-cost muon source based on the beam dump of the SHINE (Shanghai HIgh repetitioN rate XFEL and Extreme light facility) and SXFEL (Shanghai Soft X-ray Free-Electron Laser User Facility) devices. The focus of this article is on the detection of negative muons.

Firstly, the reliability of simulating the entrance of negative muons in different energy ranges is validated by comparing Geant4 with MIXE (Muon-Induced X-ray Emission) and the corresponding results. Next, based on a simple device, the material selection and thickness optimization of the Degrader for optimizing high-energy muon incidence are conducted to obtain spectral lines that are suitable for our detector resolution and ensure an adequate number of events. Subsequently, a full-process simulation based on SXFEL is performed to find the placement of the detection device that maximizes the signal-to-noise ratio, along with the corresponding μ/γ ratio used to infer muon yield from the photons received by the detector. Finally, the corresponding Data Acquisition (DAQ) system is described, along with the preparations for the next phase of SXFEL experiments.

Keywords: GEANT4 simulation, musrSim simulation, muon source, negative muon

1. Introduction

The muon is the second-generation lepton in the Standard Model. It has a mass of approximately 105.7 MeV/c^2, about 207 times greater than the electron's mass. Muons participate in electromagnetic and weak interactions and have an average lifetime of about 2.2 microseconds. They commonly decay through weak interactions into an electron, a neutrino, and an antineutrino.Muons are abundant in the universe, especially as a result of cosmic ray interactions with atomic nuclei in the Earth's atmosphere, generating a large number of atmospheric muons. Their high momentum and decay capabilities make them a source of interference in particle physics experiments, leading to the use of muon detectors in outer layers.

Muons have various applications, such as precise positioning in materials using μSR scanning techniques [3] and imaging structures like pyramids using cosmic muons [2]. They also contribute to the study of flavor physics, weak interactions, and exploring mechanisms for cosmic baryogenesis [4]. Lepton colliders focusing on muons represent the next generation, enabling higher energy scales and more precise verification of new physics.

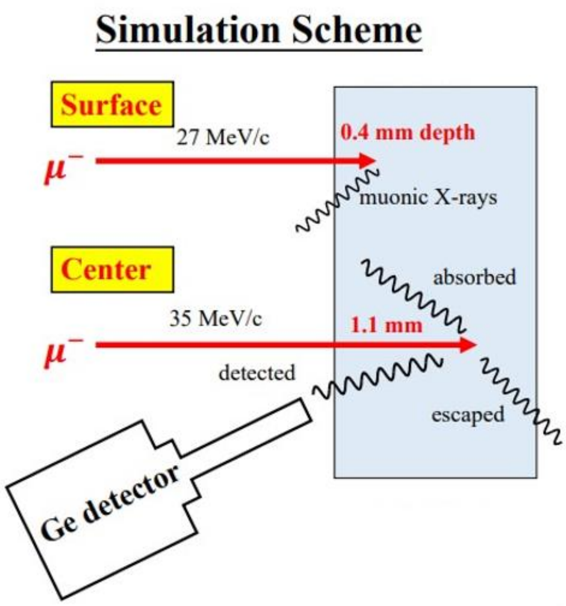
I believe the above discussion has clarified the favorable prospects of muons in particle physics experiments and engineering. However, since muons are unstable particles, there is no storage ring or similar device to store muons. Therefore, the establishment of a muon source plays a crucial role in the development of muon physics and engineering. Kim Siang's group proposed a low-cost muon source based on the disposal beam buckets of a particle accelerator. The SHINE facility under construction in the Zhangjiang High-Tech Park in Shanghai will provide an energy of 8 GeV, a charge of 100 pC, and a maximum repetition frequency of 1 MHz. The currently operating SXFEL provides parameters of 1.6 GeV energy, 500 pC charge, and a maximum repetition frequency of 10 Hz. Furthermore, constructing a muon source based on disposal beam buckets will greatly reduce the cost of the muon source.

Since SHINE is not yet completed, our group has proposed the following three-step plan: first, perform simulations based on GEANT4 to obtain preliminary parameters for target stations and beamline designs. Then, test the target stations and calibrate the corresponding detectors using electron beams at SXFEL. Finally, once SHINE is completed, use the beam from the SHINE facility to experimentally verify the feasibility of the proposed plan.In this phase, the project involves GEANT4 simulation design and experimental device debugging. The project tasks include μ- and μ+ detection, beamline design, and other aspects. This article focuses on the simulation and detector setup for μ- detection. First, the experimental principles of μ- detection are introduced. Then, the reliability of musrSim, which is used for simulation, is validated through different method for cross check. Subsequently, the optimization of absorber materials for μ- detection experiments is discussed as well as the thickness for high energy μ-. What follows is the investigation of the placement of detector aside with SXFEL and our preparation for DAQ system in preparation for following beam test hold in SXFEL and SHINE.

2. Simulation

One of the purposes of this article is to use simulation software to find a method to detect the yield of negative muon. To detect the yield of negative muon, we need to understand the properties of negative muon. Negative muon is a negatively charged, decaying particle with a mass of approximately 105.7 MeV. In the SXFEL and SHINE facilities, high-energy electrons and related secondary particles are the main background, which means that we cannot directly measure the yield of negative muon using charged particle detection methods. A clever approach is to utilize muonic X-rays to obtain information about negative muon, as shown in Figure 1.

Negative muon is a negatively charged particle, and during its passage through atomic nuclei, many negative muon particles are captured by the nuclei. During the transition of negative muon from higher energy levels to lower energy levels, muonic X-rays are emitted. In this process, muons also decay into an electron, an electron antineutrino, and a muon neutrino $\mu \rightarrow e + \overline{v}_e + v_\mu$; Subsequently, the electron antineutrino reacts with a proton, producing a positron and a neutron $\overline{v}_e + P \rightarrow n + e^+$; The annihilation of positrons and electrons generates a large number of photons with a specific energy of 511 keV which is commonly see in following result but has little effect on our analysis.



[fig 1 Muonic X-ray mechanism]

The main challenge faced by the experiment is the high background noise. Photons can originate from various sources, and bremsstrahlung radiation leads to a high deposition in the low-energy region. In reality, multiple physical processes occur simultaneously. To have sufficient precision in resolving spectral lines, a Ge detector is used in the simulation. However, in the actual experiment, a LaBr3 detector is used.

The difference between the Ge detector and the LaBr3 detector lies in the following aspects: Ge detectors are recognized as having the highest energy resolution, with a spectral broadening of only about 3%. However, they are extremely expensive. On the other hand, LaBr3 detectors have a broader energy resolution of about 7%, but they are more cost-effective. In the simulation process, we use musrSim to control the energy broadening of the incident particles to achieve a realistic effect in the detector's energy response. But the simulation have infinite energy spectrum resolution thus we can use Ge instead of LaBr3 to maintain consistence with the paper.

In the actual experiment, the only physical information we obtain is through an oscilloscope, specifically the X-ray energy spectrum and corresponding timing information. When the event rate density is high, the timing information is nearly useless. Therefore, we can only calibrate the negative muon yield based on the energy spectrum. Before further simulation, we need to validate the reliability of our simulation.

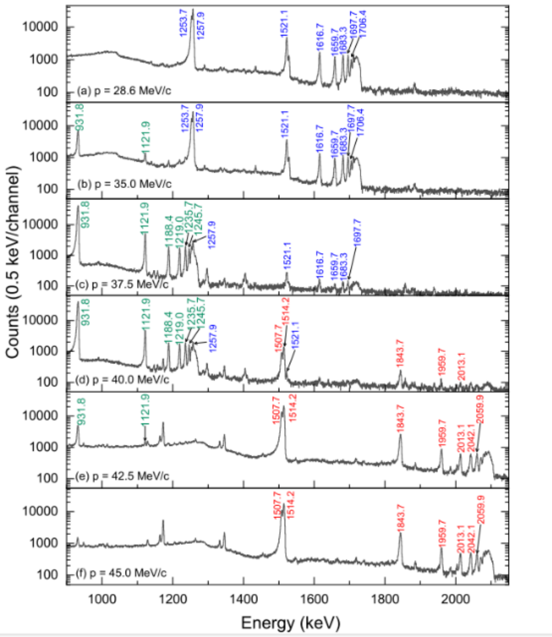
2.1 Reliability Testing of Simulation Software

Geant4 is a highly acclaimed Monte Carlo simulation software for high-energy particle experiments, and the reliability of its parameters has been validated by CERN, as referenced in article [6]. The software we are using is musrSim, which is derived from Geant4 and specifically designed for muon experiments. However, in the spirit of particle physics experimenters, we still need to verify its reliability ourselves.

2.1.1 Verification Using MIXE Results

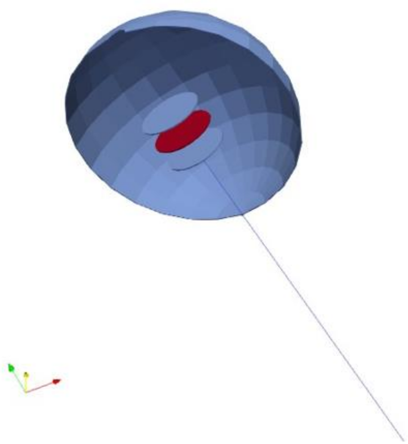
Referring to the MIXE article [5], we used a Ge detector to detect Muonic X-rays resulting from muons with different incident momenta striking the structure of the sampler, as shown in Figure 2. The muon source parameters for MIXE were $p = 20$ MeV/c with a frequency of $F \sim 1.5$ kHz and $p = 45$

MeV/c with a frequency of F ~ 60 kHz. The figure presents the results obtained from a 30-minute sampling of muons with incident momenta ranging from 28.6 MeV to 45 MeV.



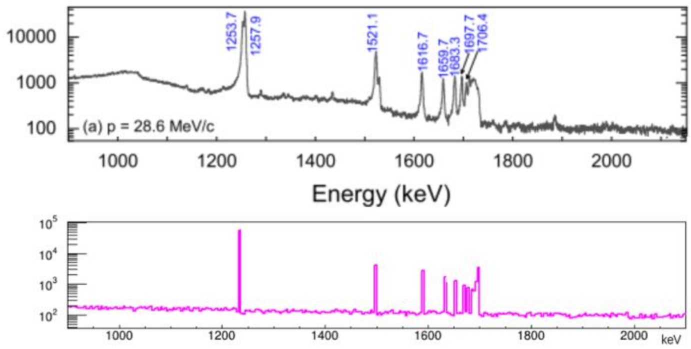
[fig 2 MIXE Results for Different Incident Momenta]

The target station setup for MIXE consists of sequentially arranged Fe, Ti, and Cu layers with the same thickness. The specific parameters are circular discs with a thickness of 500 μm and a diameter of 26.5 mm. In the image, the blue markers represent muonic X-rays from Fe, the green markers represent muonic X-rays from Ti, and the red markers represent muonic X-rays from Cu.



[fig 3 MIXE Simulated Diagram of the Target Station]

With these results, we designed a simple experiment where μ- particles with the corresponding momenta are incident on the target station described above. We used a hemispherical Ge detector to detect γ photons, as shown in Figure 3. We obtained the results for the incident of 10^7 particles and demonstrate here with momenta of 28.6 MeV as shown in Figure 4.

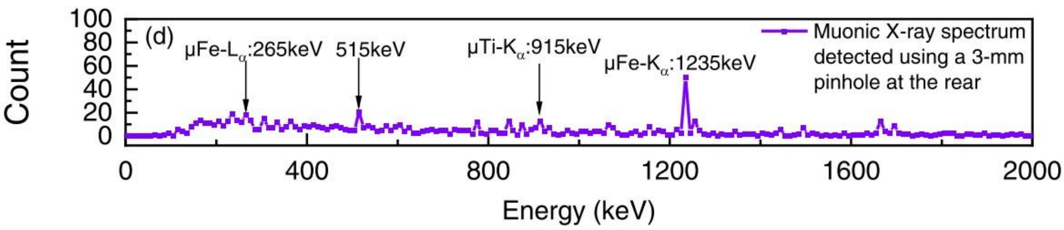


[fig 4 MIXE Target Station Simulated Results for 28.6 MeV]

Translation: It can be observed that the spectral line positions are accurate, indicating that musrSim is capable of accurately simulating muonic X-rays. It is evident that in the low-energy region, the spectral lines from Fe are prominent, while in the high-energy region, the spectral lines from Cu are prominent.

2.1.2 Verification of Low-Energy Muon Incidence

Although muonic X-rays are independent of the incident muon's energy if we ignore the significant photon attenuation due to the target, the spectral lines should remain unaffected. To avoid bremsstrahlung radiation, we did not use high-energy muons for this experiment. Instead, we studied the decay of negative muon particles at rest in Fe target materials, obtaining accurate spectral line positions. Here, we have reproduced the results from article [7], as shown in Figure 5. We used the Geant4 simulation of the μ- spectrum provided in the article to validate whether our experimental setup was correctly configured.

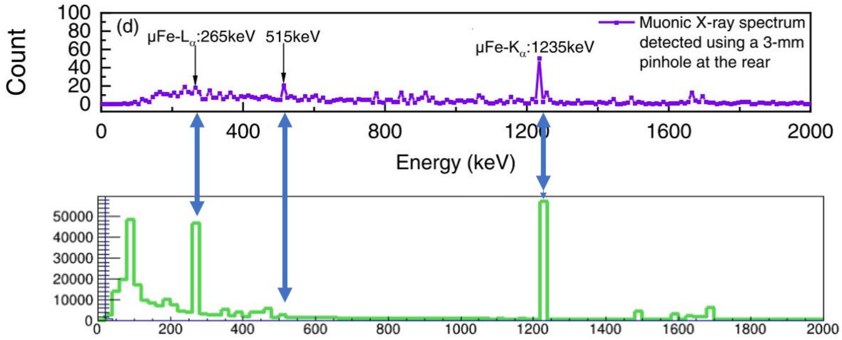


[fig 5 Geant4 Simulation of μ- Spectrum]

Our main focus of study is the K_α spectral line, which refers to the transition from the L shell to the K shell. Based on the transition mechanism, the K_α spectral line has the highest intensity. This is evident in the simulation results, where the K_α series of spectral lines exhibit the highest intensity.

In our simulation, we clearly observe accurate positions for the $\mathrm{Fe} - \mathrm{K}_\alpha$ spectral lines, as shown in Figure 6. However, it is important to note that due to different experiment settings, incident particle count being four orders of magnitude higher than in the article [7], we do not delve into the specific intensities in detail.

The accurate positioning of the $\mathrm{Fe} - \mathrm{K}_\alpha$ spectral lines in our simulation validates the reliability of our approach. Note that I mapped Ti-Ka spectrum to avoid any disturbance in the picture and the cited picture is a little different from the original.

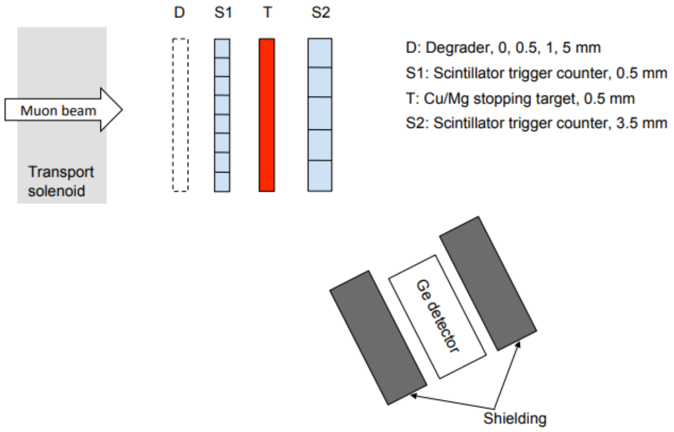


[fig 6 my result from geant4simulation]

Thus, these findings have yielded satisfactory results and also validated the reliability of musrSim.

2.2 Detector Design

After validating the reliability of musrSim, we can proceed with the design of the detector. The detector design consists of two main parts: optimizing the detector modules and conducting an analysis through a comprehensive simulation process. The simulation of the detector involves both angle optimization and material optimization. The configuration of the detector is shown in Figure 7, adopting the design principles from MuSIC(Muon Science Innovative Channel) [1].



[fig 7 MuSIC muon detecting devices]

This detector is the Muon Science Innovative Channel (MuSIC) device used for detecting positive/negative muons. Positive muons are repelled by atomic nuclei, so they do not get absorbed after passing through high-density metals. The scintillator is responsible for recording time information and is suitable for investigating the time and intensity information of low-flux positive muons. The aluminum attenuator is used to reduce background particles such as electrons and other high-energy particles. We know that μ- (negative muons) have higher momentum and strong penetrating power, so the majority of μ- particles will pass through the degrader, and a considerable number of muons will be stopped by the copper absorber. Since μ- carries a negative charge, it can be captured by atomic nuclei, undergo transitions, and emit high-energy X-rays. After being attenuated by the aluminum to reduce electron background, the X-ray signals received by the germanium detector can effectively reflect the information about the incident μ- particles.

We attempted to perform a full-process simulation near the SXFEL and SHINE devices to analyze the muon detection. However, due to high background and insufficient event numbers, we were unable to obtain conclusive results. Therefore, we opted to analyze the muon detection using a simplified device instead.

2.2.1 Degrader material choosing

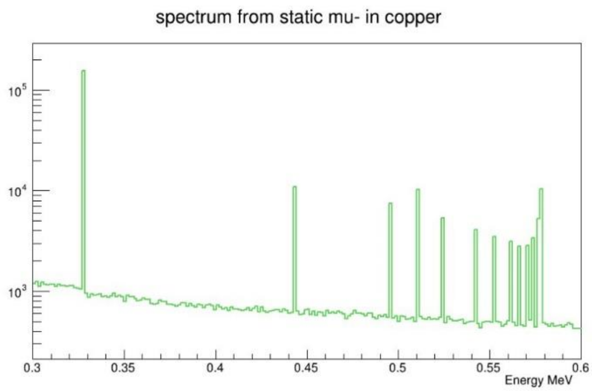
Radiation length represents the distance traveled by a particle in a material. More precisely, it is defined as the average distance over which high-energy electrons lose energy through bremsstrahlung radiation until their original energy is reduced by a factor of 1/e in the medium.

Ti, Fe, and Al are commonly used degrader and shielding materials in institutions such as MuSIC and PSI with proper radiation length. In this case, we will select the material with better performance, considering the same thickness, from these three materials to serve as our degrader.

[fig 8 Degrader material Selection Experimental Configuration]

We adopted the following configuration, as shown in Figure 8. The red component represents the germanium (Ge) detector, while the blue components consist of the copper (Cu) absorber and the degrader made of different materials. The specific setup remained consistent with the MIXE verification experiment, and the reliability of the simulation experiment was validated by comparing the results with gamma energy spectra.

In the absence of a degrader, when the negative muon particles are incident on the copper absorber, a series of spectral lines can be observed in the Ge detector, as depicted in Figure 9.

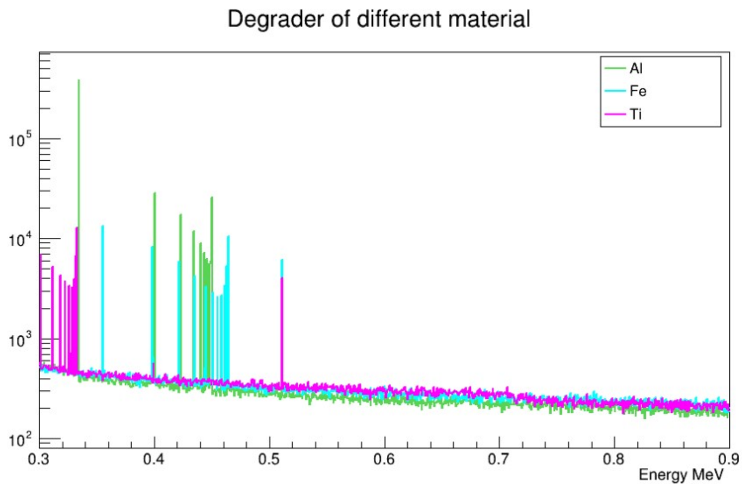


[fig 9 Spectrum Obtained from mu - Incident Copper]

After replacing different degraders, as shown in Figure 10, the green line represents the Al degrader, the pink line represents the Ti degrader, and the blue line represents the Fe degrader.

Since we need to get information from spectrum and their peaks, we need to see those peaks very clearly and have enough event rate as well. We can see the spectrum from Ti is highly compressed and bad for low resolution detector. Fe has better performance in resolution but bad in event rate. Therefore, considering the overall performance of muon energy spectrum detection and the number of events, we recommend using Al as the degrader.

Due to its higher density, Ti exhibits the highest energy absorption for muons, causing the largest shift in the spectral lines. However, considering the detector's resolution, we do not recommend using Ti as the degrader. The Fe degrader shows the sparsest spectral lines, but it can be observed that the amplitude decrease is the most significant. This might be related to the electronic structure of Fe, which makes it easier for the Fe nuclei to capture muons.

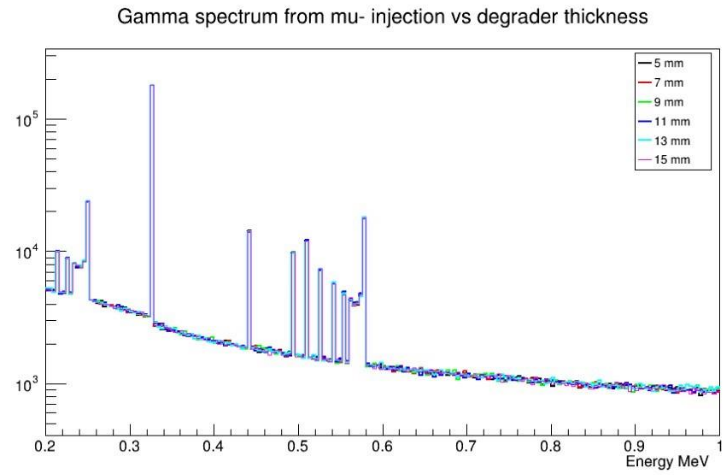


[fig 10 Spectrum Obtained from Incident negative muon on Al, Fe, and Ti]

2.2.2 Discussion on Degrader Thickness

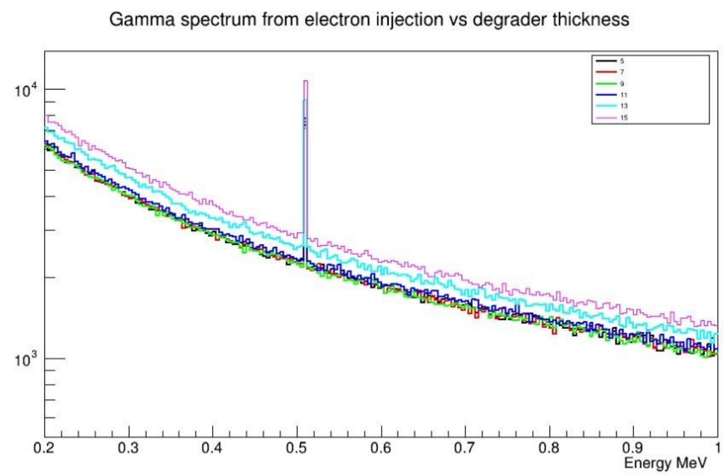
After considering the choice of the target material, we further consider the appropriate thickness of the degrader. To evaluate the thickness, we conducted simulations based on the attenuation of the electron spectrum. We performed two sets of control experiments.

In the first set, we used incident negative muon particles with an energy of 28.6 MeV and employed degraders with thicknesses ranging from 5 to 15 mm, with a 2 mm interval. The results showed that the negative muon peak did not exhibit significant changes, as depicted in Figure 11. This is because the energy is relative high comparing to the thickness changing in degrader



[fig 11 Spectrum Obtained from Incident negative muon on Al Target with Different Thicknesses]

Considering that in our real experiments, we not only have low-energy electrons passing through but also high-energy electrons as background radiation. Low-energy electrons have low penetration capability and can be blocked directly by the Al target. On the other hand, high-energy electrons generate a significant amount of bremsstrahlung radiation, which significantly increases the background level. We can consider electrons and muons to be in a mixed equilibrium state. Therefore, it is appropriate to examine the results obtained by using the same setup of a 28.6 MeV incident negative muon with Al degraders of 5-15 mm thickness at 2 mm intervals, as shown in Figure 12.

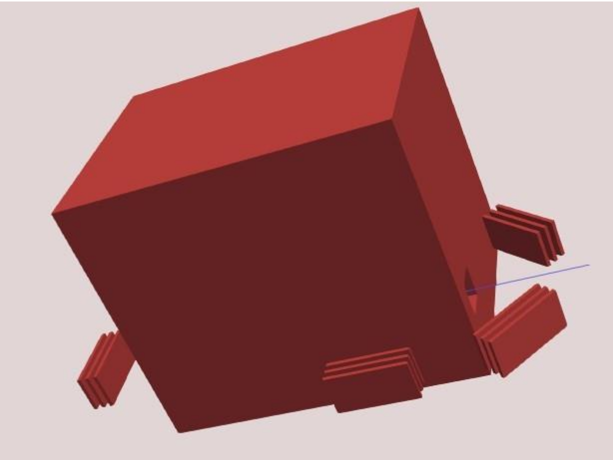


[fig 12 Spectrum of Electrons Same Momentum on Al Target with Different Thicknesses]

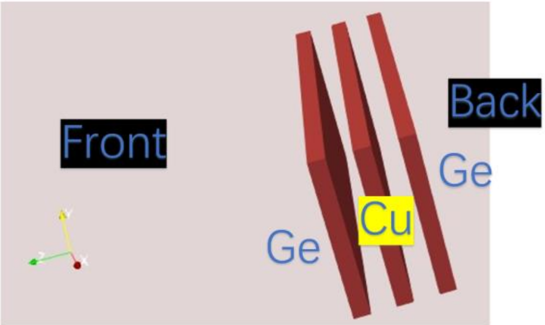
In this thickness and energy range, it can be observed that it is not advisable to set the degrader thickness too high. This is because when high-energy electrons penetrate the degrader, they emit a significant amount of bremsstrahlung radiation, which increases the background and overwhelms the data. Therefore, a thickness range of 5-9 mm appears to be optimal for high energy muon injection. Though specific thickness shall be determined by real experiment.

2.3 Full Process Simulation

We have already optimized our detector, absorber, and degrader. The next step is to consider a comprehensive simulation of the entire process. This simulation will utilize the SXFEL experimental setup and involve four detectors representing four potential detection positions. The experimental setup of SXFEL involves using a 1.6 GeV electron beam to bombard the target material. The simulation involves a total of 10^8 incident electrons. We will utilize various particles obtained from the reaction in SXFEL bulk for the corresponding simulations. In this experiment, we aim to determine the optimal placement of detectors in four orientations and determine whether the front or back position is better for placing the detectors.



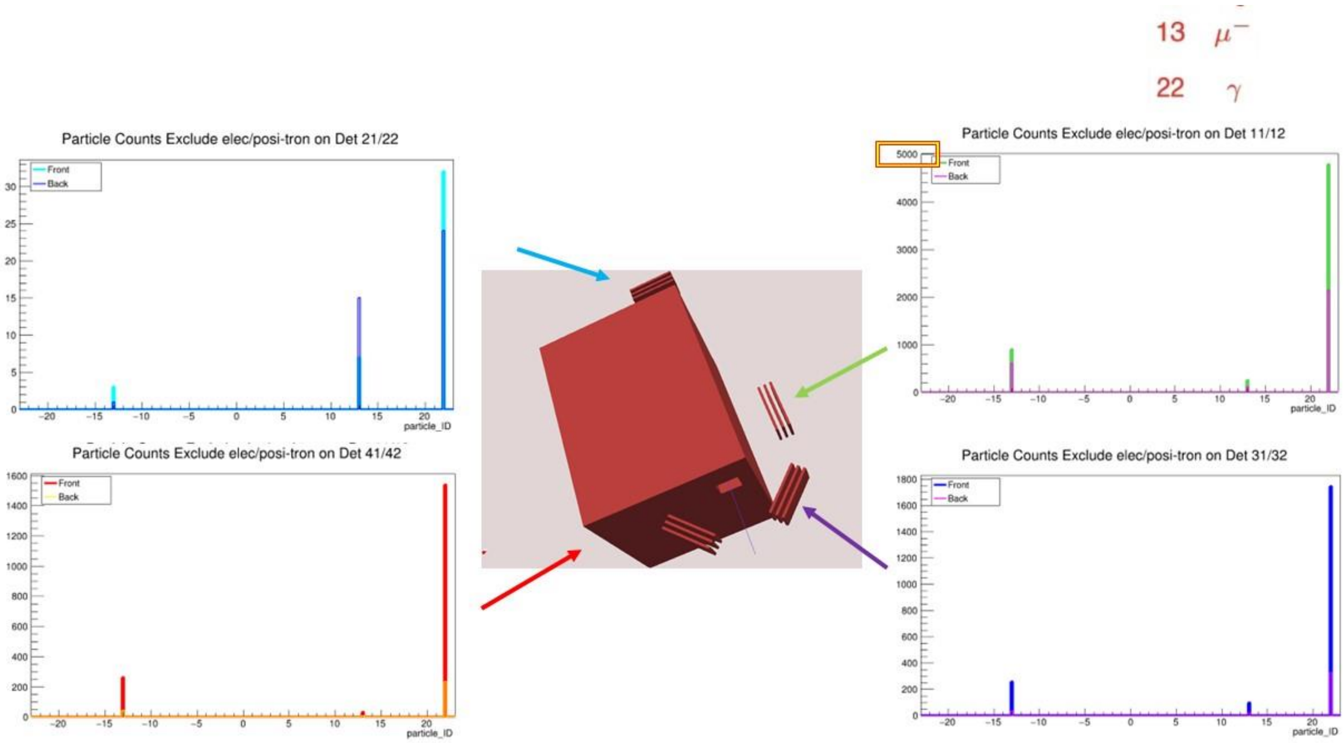
[fig 13 SXFELtarget station and detector]



[fig 14 SXFEL detector’s detail]

To improve detection efficiency and reduce space occupancy, the detector adopts a sandwich structure. It consists of a Ge detector, a Cu absorption plate, and another Ge detector. The two Ge detectors correspond to the front and back of the Cu absorption plate and are labeled as "Front" and "Back" respectively .

The results are presented below in the form of histograms (see Figure 14) showing the types and quantities of particles detected by different detectors. The received particles underwent processing, excluding the two largest backgrounds of positive and negative electrons. The horizontal axis represents the PDGid, and the main remaining particles are negative muons with PDGid=13, positive muons with PDGid=-13, and photons with PDGid=22. Each plot contains two colored spectra lines representing the possible placement positions of each detector group, namely Back and Front. The four plots represent different detector placement positions.



[fig 15 simulation result]

The optimal position for deploying the detector is determined by the highest signal-to-noise ratio at that position. The signal refers to the muon (μ-) count, and the total signal represents the number of photons. In Figure 15, each detector has two numbers. The first digit (1-4) represents four different placement positions, and the second digit (1-2) represents the front and back positions (Front and Back). The x-axis of the graph represents the PDG-ID, where 13 represents μ- and 22 represents γ. The y-axis represents the number of events, and different colors represent Front and Back.

The statistical analysis of the above results yields Figure 16, which records the number of events, the ratio of events for the same particle, the signal-to-noise ratio for the reactions, and the average signal-to-noise ratio.

- Received Events

	μ-	μ+	γ
det_11	252	906	4780
det_12	116	609	2139
det_21	7	3	32
det_22	15	1	24
det_31	97	256	1743
det_32	23	33	327
det_41	32	263	1537
det_42	7	44	234

Ratio

	μ-/γ	μ+/γ	μ/γ
det_11	0.05272	0.18954	0.24226
det_12	0.05423	0.28471	0.33894
det_21	0.21875	0.09375	0.3125
det_22	0.625	0.04167	0.66667
det_31	0.05565	0.14687	0.20252
det_32	0.07034	0.10092	0.17125
det_41	0.02082	0.17111	0.19193
det_42	0.02991	0.18803	0.21795

- Average total muon VS gamma

Front	0.21224
Back	0.24272

[fig 16 Number of events,Signal-to-noise ratio,Average signal-to-noise ratio]

By comparison, we can conclude that the side detector yields the best results. Additionally, we can determine that placing the detector at the back results in a higher signal-to-noise ratio. Specifically, the det_1 series demonstrates a signal-to-noise ratio advantage, and det_12 exhibits an even higher signal-to-noise ratio.

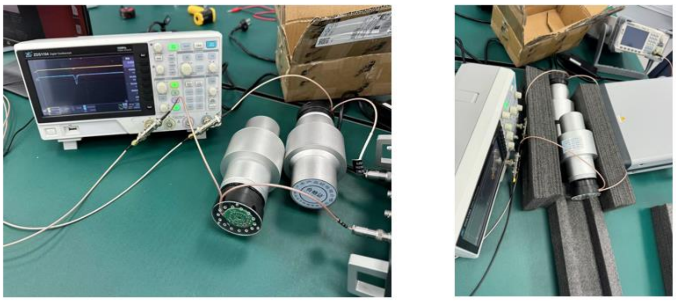
2.4 DAQ System

We have already set up the placement of the detector near SXFEL, determined the thickness and material of the detector's absorber. To further validate our simulation, we need real experiments. We can now utilize the oscilloscope and scintillator in the laboratory for preliminary data acquisition. For this purpose, we need to configure the software and hardware for the corresponding DAQ (Data Acquisition) system.

Currently, there are 8 sets of plastic scintillators and PMTs configured in the laboratory for testing cosmic muons and related experiments, as shown in Figure 17.

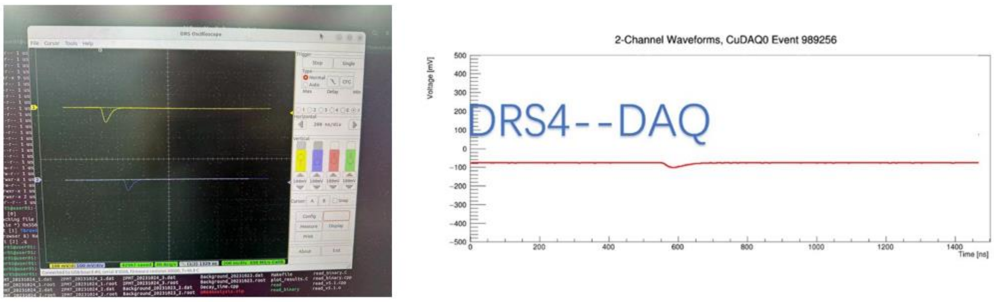
[fig 17 Plastic scintillator and PMT]

We have also installed LaBr3 detectors and conducted preliminary data acquisition experiments. The laboratory tests are shown in Figure 18.



[fig 18 LaBr3 detector]

My responsibility is to use CPP code to process binary files and convert the raw waveforms into ROOT format. ROOT is a powerful tool for high-energy particle experiments, which includes various analysis packages. By converting the waveforms into ROOT files, we can perform a series of waveform analyses, such as peak finding and Gaussian fitting, and so on... The demonstration results of converting a particular event captured by LaBr3 in oscilloscope waveforms to ROOT files are shown in fig 19



[fig 19 DRS4 raw waveform and ROOT (Left)converted waveform (Right)]

3 Discussion and foresee

This article discusses the simulation of muon yield and the design of the detector. Muons have significant importance in the next-generation particle physics experiments and engineering, leading to the proposal of a low-cost muon source based on the particle-accelerator waste beam bucket. There are three steps to validate this plan, which are currently in the stage of Geant4 simulation design and experimental device debugging. Regarding the simulation of muon yield, considering high-energy electrons and related secondary particles as the main background, we indirectly obtain the negative muon yield using muonic X-ray detection.

The experimental setup in this article refers to the MuSIC experiment and employs Geant4 and musrSim for specific simulations. The article mentions the use of Ge detectors in the simulation, while LaBr3 detectors will be used in the actual experiment. The reliability of musrSim was first verified, followed by the investigation of three commonly used materials (Ti, Fe, and Al), concluding that Al is more suitable for the Degrader material selection. Regarding the thickness, a thickness of 5-9 mm was found to be optimal for 28.6 MeV muon incidence. Subsequently, in the full simulation process, it was discovered that placing the detector sideways would yield a higher event rate and a higher signal-to-noise ratio. Therefore, in the actual experiment, it is recommended to place the detector sideways for the collection of muonic X-rays. Finally, the effectiveness of converting binary files from the DAQ system into ROOT analysis files is demonstrated.

This project identifies the challenges of SXFEL beam testing, including insufficient data, high background, and low detection efficiency. However, these issues are expected to be addressed in the laboratory once SXFEL is established. The experimental design will be further improved in the future, and the data acquisition volume will significantly increase as the experiment progresses to the SHINE test phase at 8 GeV. After determining the experimental space, the optimization of the detector's spatial position will be conducted to reduce background. We believe that these challenges will be overcome, and the research on the SHINE project is steadily advancing. We firmly believe that one day the SHINE muon source will be successfully constructed and make greater contributions to the progress of scientific research.

4 Acknowledgement

I would like to express my gratitude to Professor Kim Siang Khaw, Doctor Yusuke, and senior student Wang Jiangtao for their guidance and education in the PRP project. The weekly team meetings have greatly facilitated the rapid progress of this project. I would also like to thank my team members, Liu Fangchao and Chen Siyuan, for our collaborative efforts in solving various challenges during the experimental process. Lastly, I am grateful to my school and TDLI for providing the PRP platform, which has given me the opportunity to participate in scientific research activities. I have gained invaluable experience from this one-year research practice.

5 Reference

[1] Cook, S., D'arcy, R., Edmonds, A., Fukuda, M., Hatanaka, K., Hino, Y., ... & Yoshida, M. (2016). MuSIC: delivering the world's most intense muon beam. *arXiv preprint arXiv:1610.07850*.

[2] Morishima, K., Kuno, M., Nishio, A., Kitagawa, N., Manabe, Y., Moto, M., ... & Tayoubi, M. (2017). Discovery of a big void in Khufu’s Pyramid by observation of cosmic-ray muons. *Nature*, *552*(7685), 386-390.

[3] Hillier, A. D., Blundell, S. J., McKenzie, I., Umegaki, I., Shu, L., Wright, J. A., ... & Watanabe, I. (2022). Muon spin spectroscopy. *Nature Reviews Methods Primers*, *2*(1), 4.

[4] Bernstein, R. H., & Cooper, P. S. (2013). Charged lepton flavor violation: an experimenter’s guide. *Physics Reports*, *532*(2), 27-64.

[5] Biswas, S., Gerchow, L., Luetkens, H., Prokscha, T., Antognini, A., Berger, N., ... & Amato, A. (2022). Characterization of a continuous muon source for the Muon-Induced X-ray Emission (MIXE) Technique. *arXiv preprint arXiv:2202.03912*.

[6] Bogdanov, A. G., Burkhardt, H., Ivanchenko, V. N., Kelner, S. R., Kokoulin, R. P., Maire, M., ... & Urban, L. (2006). Geant4 simulation of production and interaction of muons. *IEEE Transactions on nuclear science*, *53*(2), 513-519.

[7] Hu, Y., Kuang, P., Li, C., Liu, F., Wu, H., Xiao, D., ... & Wei, L. (2023). Investigation of the muonic atoms distribution in materials through monic X-rays momentum simulation using Geant4. *Physica Scripta*.